

Spectral theory of first order systems: an interface between analysis and geometry.

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We consider an elliptic self-adjoint first order pseudodifferential operator acting on columns of complex-valued half-densities over a connected compact manifold without boundary. The eigenvalues of the principal symbol are assumed to be simple but no assumptions are made on their sign, so the operator is not necessarily semi-bounded. We study the following objects:

a) the propagator (time-dependent operator which solves the Cauchy problem for the dynamic equation),

b) the spectral function (sum of squares of Euclidean norms of eigenfunctions evaluated at a given point of the manifold, with summation carried out over all eigenvalues between zero and a positive λ) and

c) the counting function (number of eigenvalues between zero and a positive λ).

We derive explicit two-term asymptotic formulae for all three. For the propagator "asymptotic" is understood as asymptotic in terms of smoothness, whereas for the spectral and counting functions "asymptotic" is understood as asymptotic with respect to the parameter λ tending to plus infinity. In performing this analysis we establish that all previous publications on the subject are either incorrect or incomplete, the underlying issue being that there is simply too much differential geometry involved in the application of microlocal techniques to systems.

We then focus our attention on the special case of the massless Dirac operator in dimension 3 and provide simple spectral theoretic characterisations of this operator and corresponding action (variational functional).

[1] O.Chervova, R.J.Downes and D.Vassiliev. The spectral function of a first order system. Preprint arXiv:1204.6567v1.