A ZERO-RANGE MODEL OF A LOCALIZED STRESS ON A TECTONIC PLATE WITH A DISSIPATIVE BOUNDARY CONDITION

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Small vertical displacements η of thin elastic plates floating on a potential liquid almost non-viscous, for the slow motions, underlay filling a compact 3D domain $\Omega, \omega \in \partial\Omega$, are described by the homogeneous equation:

$$(m + \mathcal{N}\mathcal{D}_0)\,\eta_{tt} + \beta\eta_t + \Delta^2\eta + \eta \equiv \mathcal{D}\eta_{tt} + \beta\eta_t + L\eta = 0, \tag{1}$$

where $\mathcal{N}\mathcal{D}_0$ is the ω-component of the Neumann-to-Dirichlet map of the Laplacian restricted onto the orthogonal complement of boundary data on ω of constant eigenfunction of the Neumann Laplacian in Ω

$$\mathcal{N}\mathcal{D}_0 \equiv \sum_{l>0} \frac{\varphi_l^\omega\rangle\ \langle\varphi_l^\omega}{\lambda_l}. \tag{2}$$

The small parameter β describes the liquid dissipation and the bi-harmonic operator $L \equiv \Delta^2 + I$ is supplied with appropriate dissipative boundary conditions characterized by matrices Γ, Γ^+ at the boundaries $\partial\omega \equiv \gamma$. The collisions of the plates at the comparatively small active zones on the boundaries can be modeled by the corresponding zero-range interactions, which are defined by hermitian or dissipative 3×3 matrices $\kappa, , \Im\kappa \geq 0$. The zero-range technique for the dissipative operators is developed based on Krein formula for the resolvent of the corresponding selfadjoint dilatation. This technique yields a formula for the corresponding characteristic function

$$S_\kappa = I - i\Gamma\frac{I}{A_\kappa^+ - \frac{i}{2}\,\Gamma^+\Gamma - pI}\Gamma^+,$$

containing the characteristics Γ, Γ^+ of the dissipative boundary conditions on $\gamma \equiv \partial\omega$ of the biharmonic operator on the plate, and the operator extension matrix κ, which characterizes the localized stress. It can be interpreted as a Saint-Venant parameter of the collision of the neighboring plates in the active zone.

References

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